Award Paper

I Don't Understand Part b or How You Find the Answer!: The Development of a Student Response Feedback Framework for Evaluating Student Approaches to Unfamiliar Application Problems

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Abstract

The purpose of this study was to investigate the patterns of problem solving behaviour displayed by Year 11 students, when attempting to use recently studied mathematics on problems set in unfamiliar, life-related contexts. The aim is to develop a framework to guide teachers in the type of feedback they give to students. Patterns of behaviour within the categories of engagement, planning and monitoring, use of heuristic strategies and verification skills, knowledge of facts and procedures, and beliefs were inferred from a study of written responses to four questions A Student Response Feedback Framework is proposed and suggestions are offered for a range of uses for this instrument.

The Problem

The ability to use mathematical knowledge in contexts that are different to those in which it was acquired, in particular life-related situations, has been a focus of mathematics education for at least the past decade. The findings of research programs aimed at investigating the difficulties associated with Problem Solving and Transfer, and the way in which students' performance in these areas may be enhanced, have proved inconclusive and at times contradictory. Further, little research has been conducted into how teachers might assess problem solving skills (Charles and Silver 1989), and consequently, provide students with meaningful feedback about their performance on problem solving tasks. As noted by Silver and Kilpatrick (1989), there has been little reflection on how the testing of problem solving operates and "current tests are not generally helpful to teachers on how problem-solving instruction should proceed" (p. 179).

The aim of this study was to investigated the mathematical problem solving behaviour exhibited by students on life-related tasks in order to develop a framework, related to theories of human learning, for providing students with meaningful feedback.

Synthesis of Research Evidence

Current interest in teaching students to use mathematics in life-related situations is reflected in both curriculum development and in research (e.g. Burkhardt 1981; Lesh and Akerstrom 1982). This, in part, stems from a disenchantment with traditional pedagogical practice which, most commonly, revolves around an explanation followed by an example followed by drill and practice (Romberg and Carpenter 1986). It is believed that this no longer constitutes a complete mathematical experience.

Despite increased efforts to relate mathematics studied in classrooms to unfamiliar and life-related contexts, students still experience difficulty using mathematics in non-routine situations. Many of these efforts have focused on an approach to problem solving outlined by Polya (1945, 1954, 1981). The results of research, however, have been inconsistent and inconclusive (Schoenfeld 1985). This has challenged educators to look beyond simple Polyarian models of problem solving instruction based on training in general problem solving strategies alone.

In recent times knowledge about the way we solve problems has been supplemented by research into the way we think and learn. In particular, work in the fields of cognitive science (Schoenfeld 1987), and affects (McLeod 1992) has provided greater insight into the process associated with problem solving, and how these processes are influenced by belief systems.

Significance of Knowledge Structures

According to Schoenfeld (1985) an individual must be familiar with knowledge relevant to the content domain in which a problem is set. Because this represents a vast amount of information and because there are limitations on the mechanisms used to utilise memory, issues of how information is represented and organised in memory and how it is accessed are important in understanding the role of knowledge structures in problem solving.

The organisation of, and access to, knowledge is believed to be based on complex knowledge structures (Davis 1984; Silver 1987) known variously as schemas (Skemp 1971) or frames (Davis 1984). It seems many problems are solved by accessing problem specific schemas. The more developed and rich those schemas, within a specific content domain, as in the case of an expert, the more likely a problem can be solved semi-autonomously (Schoenfeld 1990). The work of Hinsley, Hayes and Simon (1977) suggests that mathematical problems are solved via the instantiation of schema related to specific problem types. Thus individuals tend to categorise problems on the basis of their underlying mathematical structure, and then solve them using routine procedures developed for use with that particular category.

The role of a representation is vital to the problem solving process in Davis' (1984) cyclic model. This is characterised by an active and interactive attempt to match the internal representation of a problem with knowledge structures which already exist in memory. Initial representations are gradually developed through the input of further information drawn from the task or from associated knowledge structures that already exist in memory. The matching/development cycle ceases when a match acceptable to the problem solver is achieved. One source of difficulty in the process of forming a functional representation is the translating of the problem representation from one form to another. This is a crucial ability in the problem solving process (Lesh, Post and Behr 1987) particularly if the problems are set in real world contexts (Janvier 1990). Thus the initial representation of a problem is critical to the problem solving process as it largely determines the of actions which follow.

The existence of well developed and organised knowledge structures may not be enough however, when an individual is faced with what to them is an unfamiliar mathematical situation. In this case expert problem solvers also employ a wide range of strategies (Schoenfeld 1992) which enable them to cope with problems beyond the routine.

Meta-Processes

A number of researchers (e.g.Flavell 1979; Garofalo and Lester 1985) have stressed the importance of self-regulatory behaviour in the problem solving process. "Metacognition" is the term used to describe an individual's conscious monitoring and control activity during a problem solving episode. Effective monitoring and control skills allow the problem solver to make efficient use of the cognitive resources (knowledge structures) and strategies(heuristics) at their disposal (Silver 1982; Schoenfeld 1992).

Despite the importance of managerial function to problem solving endeavours, it appears that students, in general, do not display notable competence in these skills. (Schoenfeld 1983, Garofalo and Lester 1985). It appears, however, that there is good reason to be optimistic about the potential for teaching these skills. Lesh and Akerstrom (1982)and Schoenfeld (1983, 1985), for example, found that important applied problem solving processes are teachable and do improve everyday problem solving capabilities although, at best, are a long term objective. This is supported by Silver (1982) who claims many of Polya's heuristic suggestions can be regarded as metacognitive prompts that can act as powerful factors in determining problem solving behaviour. These prompts can catalyse the activation of schemas relevant to the problem solving situation providing the problem solver with access to appropriate facts, routine procedures and further strategies. However, instruction in the use of metaconitive skills is most effective when it take place in a domain specific context (Lester, Garofalo and Kroll 1989) and is fine grained enough to be prescriptive rather that descriptive (Schoenfeld 1985).

Beliefs

Although, until recently, there has been limited interest in affective influences that impinge upon problem solving activity, there are now a number of researchers that argue for the consideration of such effects in any evaluation of problem solving performance (e.g. McLeod 1985; Schoenfeld 1989). Silver (1982) comments that belief systems may help to explain: the selection of strategies; the degree to which a student will persist with a problem or strategy; and the depth of the feelings of satisfaction an individual experiences after a problem solving encounter. McLeod (1992) indicates that the feelings associated with knowledge domains actually become embedded in the personal schemas of an individual. Thus the feelings associated with past performances in a domain will be invoked whenever an individual is asked to operate in that domain.

An elaboration of the origins of these beliefs are beyond the scope of this paper but there exists a growing body of research that posits these systems grow out of what students perceive as their teachers' beliefs about the nature of mathematics, and from implicit messages about mathematics from the way they are taught (e.g. Thompson 1985). Such perceptions can only serve to emphasise the importance of social context in the development of problem solving ability.

Methodology

Support for greater use of naturalistic methodologies in research in mathematics education has come from, among others, Lester (1985), Kilpatrick (1981), and Eisenhart (1988), who argue that holistic approaches are necessary when a researcher is attempting to investigate situations which are susceptible to the influence of a large number of interactive variables, as is the case in studies of mathematical problem solving in classroom settings. While the generalizablility of individual pieces of qualitative research is limited a mass of such studies make it possible to identify trends and tendencies which can contribute to the support and generation of theory. Stenhouse (1975) describes this process:

> *If teachers report their own work in such a tradition, case studies will accumulate, just as they do in medicine. Professional research workers will have to master this material and scrutinise it for general trends. It is out of this synthetic task that general propositional theory can be developed.* (p.157)

The principal aim of this study was to develop a framework to guide teachers in providing structured feedback to students. Since the pressure of real classrooms means that the vast majority of such information is in the form of written responses to problems attempted in both formal and informal classroom environments, the data collected consists of written attempts to solve application problems.

The participants in this study consisted of two teachers working with 47 students from a Catholic co-educational secondary school. The students formed two classes studying the first year of a two year secondary school senior course, Mathematics I.

The style of instruction experienced by both classes was conventional, being teacher directed and following a teach and examine model. As part of this program students received instruction in solving mathematics problems set in life-related contexts. Students were presented with an outline of problem solving strategies and associated issues such as monitoring, control and beliefs were discussed. These are summarised in the student strategy guide (Appendix 1) which is a synthesis of approaches proposed by Mason, Burton and Stacey (1982) and Burkhardt (1981). Students were encouraged to use this guide during discussions of problem solving skills and when working on practice examples. Instruction on how to work on problem solving tasks and practice items was provided on a regular basis (at least every two weeks). Initially problems were set in contexts that required the use of mathematical concepts students were deemed to have already mastered, as the aim was to focus principally on problem solving skills. The mathematics in successive questions became progressively more challenging until the techniques required to solve questions included those recently studied in the content of the course work.

The major source of feedback to the students during the teaching program consisted of verbal comments on class activities. These comments were guided by the advice of Schoenfeld (1985), Mason et. al.(1982) and Polya (1945) on how to approach novel mathematical problem situations and tended to fall into four main categories: how to go about beginning a problem; what to do when you get stuck; recognition of wrong turns and dead ends; and the verification of possible solutions. Comments were aimed at prompting the problem solver to pay attention to metacognitive processes such as the monitoring of their work. Students were encouraged to record the thinking and reasoning behind decisions they made while attempting to solve problems. This included indicating if they believed they were on the wrong track and writing down why before they attempted a new direction. This was done, in part, to train students to verbalise their thought processes for the purposes of this study, but also as a method of focusing the students attention on self-monitoring and evaluation behaviours.

The assessment procedures that generated the data in this study were formative instruments aimed at assessing student learning and process capacities in the context of the course being studied. The instruments used to assess achievement in process objectives were administered as traditional pen and paper tests six times during the period of this study at approximately six week intervals. The specific data have been drawn from students' responses to four of these questions.

Problem 1

The table below has been copied from a shoe manufacturers size comparison chart. It shows the size of a shoe compared with its length (measured in millimetres), for shoe sizes from size 1 to size 8.

Shoe size	1	2	3	4	5	6	7	8
Length of shoe (mm)	220.1	228.6	237.1	245.5	254.0	262.5	270.9	279.4

1. Draw a graph to represent this information.
2. Find a relationship between shoe size and length in millimetres and express it:
(a) in words
(b) as a mathematical formula
3. Shoe sizes are available in half sizes, e.g., size $2\frac{1}{2}$ is the size between size 2 and size 3. What would be the approximate length of a person's foot if they took a size $4\frac{1}{2}$ shoe?
4. What shoe size would a person with a foot 261.2 mm in length require?

Problem 2

An influenza epidemic has hit a city and it is estimated that the rate of change of people without influenza with respect to time is given by:

$$\frac{dW}{dt}=400t-12000$$

where W is the number of people without influenza and t is the number of days the epidemic is in progress.

1. Find an equation W as a function of t if the number of people without influenza before the epidemic is given by $W(0)=500000$.
2. Find the number of people without influenza 30 days after the start of the epidemic.

Problem 3

Outside the central business zone, the average number of people per square kilometre who live a certain distance from the centre of the city has been shown to follow this law approximately:

$$y=A\ 10^{-bx}$$

where: y is the number of people per square kilometre
x is the distance from the centre of the city in kilometres
A and B are constants for any one city at any one time
In Sydney in 1947 the following information was collected and found to obey this law.

Distance from the centre	3	10
Number of people per square kilometre	14200	2500

Find:
1. the value of b.
2. the number of people per square kilometre who would have been resident in the centre of Sydney in 1947 by using the above law.

Problem 4

The security grills that are designed for houses are generally rectangular in shape. This question concerns a rectangular security grill with 3 bars in it as illustrated below:

In constructing such a security grill a company worked out that it could maximise its profit by using 9 metres of steel for each grill.

1. If the height of each grill is x metres, find an expression for the length of each grill as a function of x.
2. Find the maximum area that can be covered by a grill of this type. What are the dimensions of this grill?

These problems were chosen according to the following criteria: (a) problems were based on mathematical ideas previously introduced to students during their current course; (b) problems were set in unfamiliar, real-life contexts; (c) problems were of sufficient complexity to require students to use external representations in their attempts to solve the problems. During tests students received no assistance of a mathematical nature, only receiving help with the explanation of the meaning of specific words and encouragement when they appeared to be stuck.

Procedure Used to Identify Patterns Of Student Behaviour

A summary of the categories used to guide the search for patterns of problem solving behaviour within students' attempts to solve the problems appears in Figure 1. This framework represents a synthesis of the cognitive/metacognitive frameworks of Garofalo and Lester (1985), Schoenfeld (1985), and Biggs and Collis (1982).

These categories were used to guide the search for regularities in problem solving behaviour in student scripts. Each student script was examined and, where possible, a written comment on the student's performance in each categories was recorded. Comments were based on the two components of the students' scripts: (a) their attempts to solve the problems; and (b) their written descriptions of what they were attempting to do. From these comments clusters of response types within categories emerged.

Figure 1 Script analysis framework

ENGAGEMENT	(a) Problem is read and interpreted (b) Goals and givens established (c) Goals and givens represented symbolically
EXECUTIVE BEHAVIOURS	(a) Planning - Selection of strategies to aid in exploring and understanding the problem - Identification of goals and sub-goals - Global and local planning - Selection of strategies to carry out global and local plans. (b) Monitoring - "On-line" monitoring of progress through a problem - Recognition of potentially fruitful solution pathways - Recognition of solution pathways that will potentially lead to dead ends. - Coordination of the transitions between phases of problem solving such as analysing, exploring, planning, implementing and verifying (c) Heuristic strategies - Comprehending problem statements - Organising information or data - Executing plans - Planning solution attempts - Checking results (d) Verification skills (i) Evaluation of orientation and organisation - Adequacy of representation - Adequacy of organisational decisions - Consistency of local plans with global plans - Consistency of global plans with goals (ii) Evaluation of execution

EXECUTIVE BEHAVIOURS	- Adequacy of performance of actions - Consistency of actions with plans - Consistency of local results with plans and problem conditions - Consistency of final results with problem conditions
RESOURCES	(a) Informal and intuitive knowledge about the domain
	(b) Facts, definitions and the like
	(c) Algorithmic procedures
	(d) Routine procedures
	(e) Relevant competencies
	(f) Knowledge about the rules of discourse in the domain
BELIEFS	(a) Persistence of effort or lack there of
	(b) Effects of emotional state, e.g. frustration
	(c) Effects on the selection of strategies and resources

Results

Clusters of responses were classified into the categories presented in the **Student Response Feedback Framework** (Figure 2). Not all decisions about the categorisation of some aspect of a performance were clear cut, grey areas were certainly in evidence, and thus decisions are not free of subjective judgments. For example, determining whether a student failed to attempt a question because of a lack of confidence or because of a lack of factual knowledge in a particular field sometimes cannot be determined. In some cases such judgments can only be made by an observer who is familiar with the work of that individual student. In the main, however, scripts fell clearly into the categories contained in Figure 2. As can be observed, twenty-three cells are represented in the framework across the six dimensions of problem solving activity. In general decreasing competence is represented by left to right movement across the rows.

Figure 2

STUDENT RESPONSE FEEDBACK FRAMEWORK

	ENGAGEMENT	Well formed representation	Incomplete representation	Faulty representation	No representation	
EXECUTIVE BEHAVIOURS	PLANNING AND MONITORING	Solution planned and monitored effectively.	Solution planned but obstructed. Monitoring ensures a controlled investigation.	Solution planned but obstructed. An inappropriate pathway is pursued with little or no evidence of monitoring.	Failure to pursue a promising option	No planning.
EXECUTIVE BEHAVIOURS	HEURISTIC SKILLS	Heuristics purosefully used as a direct aid to solving the problem.	Heuristics employed as a means of clarifying the problem statement.	No heuristics were used.		
EXECUTIVE BEHAVIOURS	VERIFICATION SKILLS	A verification procedure is used to successfully test for the validity of a solution.	A verification process is used to identify a problem with a solution attempt.	A verification procedure is not used		
RESOURCES	FACTS AND PROCEDURES	A complete knowledge of facts and procedures is exhibited.	A knowledge of facts and procedues is in evidence but minor errors have occurred.	Some knowledge of the necessary facts and procedures appears to exist but there are significant faults exhibited in execution.	Knowledge of the relevant facts and procedures have not been demonstrated.	
	BELIEFS	Self confidence and determination is evident in the persistence of the effort.	A lack of self confidence and persistence are evident.	Default behaviours indicate that belief systems have a strong negative influence on executive priorities.	Self confidence and determination appear to be lacking as no attempt is made.	

As it is beyond the length restriction of this paper to discuss comprehensively and illustrate all response types in the framework, selected examples from the Engagement category only will be presented. The examples will illustrate response types within this category, and indicate the characteristics of varying response quality as defined within Figure 2.

The Engagement category relates to the student's first contact with the problem situation. At this point an initial internal representation is developed from the information contained in the problem. The initial completeness of this representation is variable and was found to be crucial to the directions students choose in the search for solutions. This is consistent with the findings of Janvier (1990) and Lester, Garofalo and Kroll (1989) who have stressed the importance of initial representations to problem solving performance. The responses fell into four sub-categories.

(a) *Well Formed Representation* allows students to operate at a level that approaches expert performance. The complete nature of the initial representation makes it possible for the student to activate and instantiate an appropriate schema so effectively that the student appears to function at an almost automatic level. Such a representation can make a problem appear trivial to such a student, even though it may appear to be quite difficult to others in the group (Silver 1979). The script below is an example of student F.N. working on problem 2.

$$\frac{dW}{dt} = 400t - 12000$$

$W = \dfrac{400t^2}{2} - 12000t + C$ (anti derivative:)

$W = 200t^2 - 12000t + C.$

i). $W(0) = 500\,000$

$W(t) = 200t^2 - 12\,000t + C$

$W(0) = 0$ (sub. in for $t = 0$)

$\therefore C = 500\,000$

$\therefore W(t) = 200t^2 - 12\,000t + 500\,000$

ii). $W(t) = 200t^2 - 12000t + 500\,000$

$W(30) = 200 \times 30^2 - 12000 \times 30 + 500\,000$

$= 180\,000 - 360\,000 + 500\,000$

$= 320\,000$ people without influenza

... 30 days after the start of the epidemic.

(b) An *Incomplete Representation* is one that while relevant is not sufficiently developed to lead directly to a solution. Relevant schemas appear to be activated initially but executive behaviours are necessary to develop the representation to the point where a solution is accessible. This means that the role of a well developed resource base is vital to the progress toward this solution. Below is an excerpt from a paper submitted by K.W. as she attempted to solve problem 4.

1. ~~$L = 2(a+b)$~~ $P = 2(l+b)$ (Perimeter of a Rect.)
~~$P = 2(x+l)$~~ $P = 2(x+y)$
Perimeter = 9
But there are three bars so:-
$5x + 2y = 9$
you can't have 2 unknowns!
$2y = 9 - 5x$
$y = \frac{9-5x}{2}$ ①
$9 = 2\left(x + \frac{9-5x}{2}\right)$
An expression with only one unknown.
$f(x) = 2\left(x + \frac{9-5x}{2}\right) - 9$

2. Find x.
$2 \cdot 9 = 2x + 18 - 10x$
$18 = 2x + 18 - 10x$
$10x = 2x + 18 - 18$
$10x = 2x$
$10x - 2x = 0$
$8x = 0$
$x = 0$

answer was zero, so I must have done something wrong because you can't have a security grill 0 metres ~~hei~~ high
Trying again over page

In this attempt she appears to identify the type of problem she is dealing with (recognising that it is an optimisation problem, and that single variable differential calculus is required) and begins the question, although it seems she is not completely sure of how to proceed toward solution. She has to represent the problem externally and develop her ideas further from this point in order to make additional progress.

(c) *Faulty Representations* are evident when students attempt to solve problems by methods that are totally inappropriate. It appears that these students are unable to activate a relevant schema (if it exists), possibly because they have focused on surface features of the problem (Lester et al. 1989), and the representation that was formed is an inappropriate one. The example used here is from J.F.'s work on problem 2.

$$\frac{dw}{dt} = 400t - 12000$$

$$\frac{\text{people}}{\text{days}} = 400 \text{ days} - 12000$$

$$\frac{dw}{dt} = 400t - 12000 -$$

$$F(t) = \frac{500000}{400t}$$

=

the total people is 500000 + 12000
the number of days is 400 ~~d~~
f(w) ~~62000~~ ~~512000~~/~~400~~ 12000/500000 × 100
~~1~~ 2.4%

12000 out of 500000 is 2.4%
so out of the 100% 2.4 got the influenza in 400 days
w(0) = 97.6 ~~%~~ % haven't got it. in 400 days

so 12000/400 to get amount of people who got it a day from the start

ii) is 30 a day. × 30 for 30 days
is = 30 × 30
= 900
∴ so 900 people got it in the first 30 days

J.F. has not recognised that the solution to the problem requires the use of calculus techniques. His inability to form even a partially correct representation of the problem has forced him into exploring the problem by examining a variety of irrelevant options. It appears likely the reason for the activation of knowledge schemas so unrelated to the problem at hand is related to J.F.'s belief system about mathematics.

(d) *No Representation* was apparent when students were unable to provide any meaningful response to a problem. Typical of this cluster are blank response sheets, or a restatement of the problem and nothing more. L.S. in response to problem 3 has merely restated the problem and made an obvious substitution of variables into the equations contained in the problem statement. There is no evidence that she is able to conceptualise a method of attack for this problem or that any schema relevant to the solution of this problem are activated.

$$y = A\,10^{-bx}$$

y = people
x = distances
A + B - constants

Sydney
$$14\,200 = A\,10^{-b3}$$
$$14200 = A\,10^{-3b}$$

$$y = A(10^{-b})^{x}$$
$$14\,200 - A\,(10^{-b})^{3}$$
$$14200 - A\,10^{-3b}$$

$$2500 - A\,10^{-10b}$$
$$2500 = A\,(10^{-10})^{b}$$

Responses indicated that Engagement is a critical phase in the problem solving process as the outcomes appear to strongly influence subsequent student action. This is in agreement with the observations of Lester, Garofalo and Kroll (1989)

The four sub-categories have a degree of correspondence with the four levels of orientation described by Lester et al. (1989). The first two sub-categories describe responses in which the meaning of a question appears to be understood allowing students to develop representations based on the underlying structure of the problem. It appears that these initial representations permitted students to access general problem schemas thus initiating the cyclic process of representation development, retrieval and mapping described by Davis (1984). In the first sub-category students appeared to develop a well formed representation which incorporated all the important features of a problem while at the same time recognising and excluding all non-essential features and/or distracters. This allows the problem to be categorised and relevant schema accessed as described by Hinsky et al (1977). The second group of students developed initial representations that contained a minor fault but still ascertained the general meaning of the problem. It would seem that students' representations, in this case, while not fully developed, contained enough information to access schemas generally related to the topic area but not detailed enough to produce a mapping that led directly to a solution.

The third and fourth categories of response represent the work of students who failed to understand a problem at any better than a surface level. These students had difficulty in developing a valid representation of a problem. This may have been because of difficulties in translating a written problem representation to a mathematical model (Lesh, Post and Behr (1987). This difficulty is particularly relevant to life-related contexts (Janvier 1990).

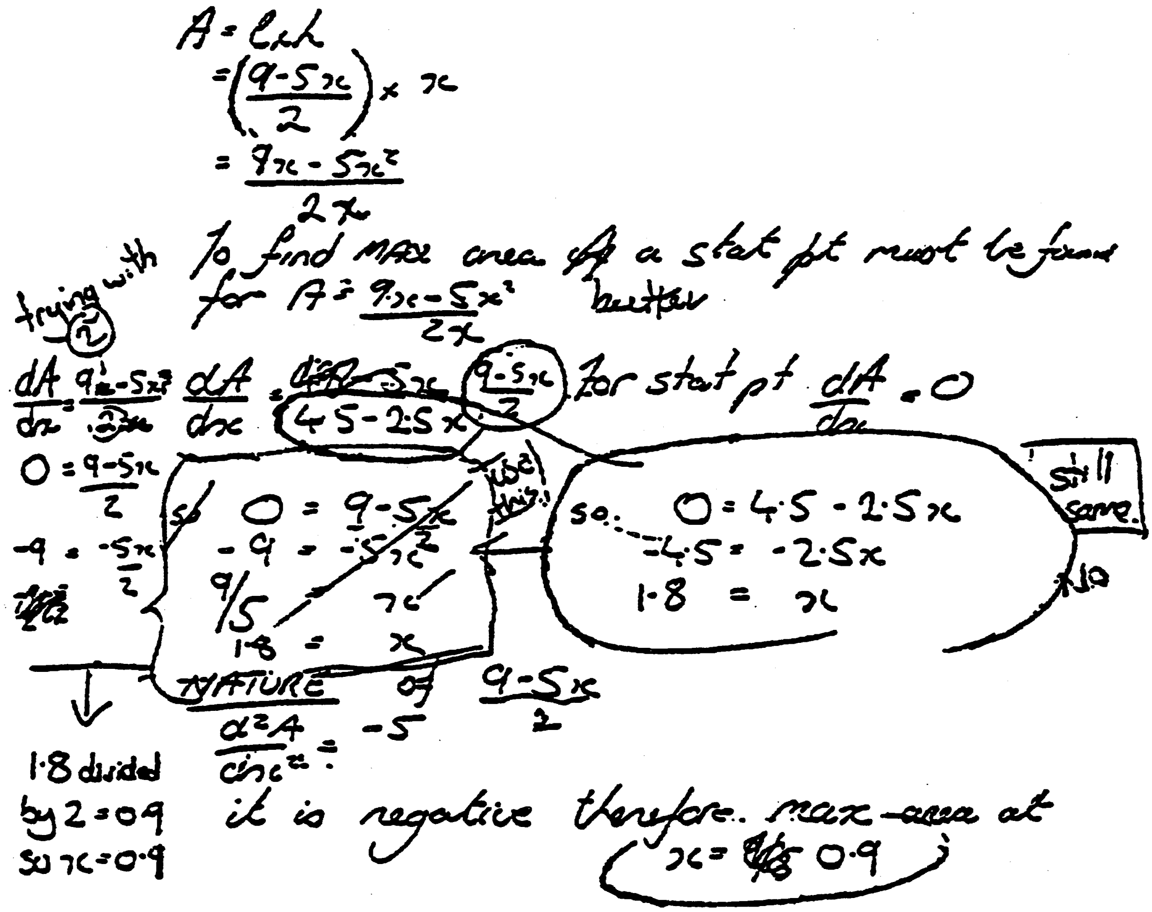
Similar evidence is available for the existence of the other categories presented in the framework. The author believes the framework to be trustworthy because of the following factors. Firstly, although general principles were agreed upon by the two teachers and informal discussions held throughout the period of instruction, no attempt was made to standardise instructional approaches. Despite the potential for localised variation in response types the clusters identified within categories were consistent across both classes of students. Secondly, response clusters within categories were consistent across different problems, even though the problems used to generate the students' responses were based on different areas of content. Thirdly, the non-interventionist approach taken when students attempted to solve problems ensured that particular responses were not the result of a form of questioning or prompting, as can be a danger in interview based studies. The responses were entirely the products of individual students responding in a free (given the constraints of formal assessment) situation.

Implications for Instruction

The study found that it was possible to identify characteristic response types from students' written scripts and relate them to theories of human learning. Some of these have been illustrated in the previous section.

Structured information about a student's patterns of problem solving behaviour can provide the teacher with valuable information for the purpose of remediation. A particular weakness in a student's performance can be identified and brought to the student's attention and a course of action, aimed at improving that aspect of problem solving behaviour, developed. Detailed knowledge about personal performance on problem solving tasks provides students with an idea about where to best direct their efforts. Further, providing students with the type of detailed information outlined in this framework, especially in conjunction with other sources of information (eg teacher observations in informal situations or performance on assignment based tasks), would draw specific attention to metacognitive processes and the role they play in problem solving.

The script which appears below is an excerpt from an attempt, by C.S., to solve problem 4. The rest of this script indicates she had a clear global picture of what to do to solve the problem, however, because of errors in multiplying by x (line 3), and in finding the first derivative (margin), she was unable to realise the potential of her plan. In addition C.S. has not made use of verification techniques which may have allowed her to detect errors. By making use of the framework a teacher is in a position to provide encouraging feedback about the student's performance in engagement and planning phases but also be able to offer remedial assistance in the knowledge domains associated with recorded errors.



Most importantly, the framework provides teachers and students with the basis of a language which might allow them to negotiate shared meanings, enabling them to communicate to each other information about problem solving performance. This will provide the student specific information about how to improve on his/her performance. For example, if a student is providing solutions with unreasonable results or failing to detect errors in facts or use of procedures, as is the case in the student script above, a teacher can encourage the student to make better use of verification techniques. In order to make use of this advice a student needs to develop detailed prescriptive knowledge about the use of these techniques (Schoenfeld 1985). By providing opportunities for the student to work on appropriate problems and feedback based on the framework, such ideas can be targeted and discussed creating a forum in which an understanding about the use of verification techniques can be developed.

The framework can also provide the basis for a process of assessment in courses which identify problem solving as an important mathematical activity. By providing students with detailed and specific criteria teachers are able to provide students with targets, made public in the framework, of what constitutes success in problem solving performance. A record of performance, profiled over time, then not only document the strengths and weaknesses of individual students but also, considered in its entirety, provides a map of a student's global performance. Such an approach directs students' and teachers' attentions to the process of problem solving rather than the product. If it is the process that is valued then perhaps this is where the thrust of assessment should also lie.

Any successful use of this framework is contingent upon students' willingness to participate in a progam in which it might be used and their confidence in the framework's potential to improve their performance on problem solving tasks. Thus time would be wisely spent in discussing and dealing with aspects of belief systems that influence problem solving performance.

References

Biggs, J., and Collis, K. (1982). Evaluating the Quality of Learning: The SOLO Taxonomy (Structure of the Observed Learning Outcome). New York: Academic Press.

Charles, R.I. and Silver, E.A. (1989). Preface in Charles, R.I. and Silver, E.A. (Eds), The teaching and assessing of mathematical problem solving (pp. v-vi). Hillsdale, NJ: Lawrence Erlbaum.

Burkhardt, H. (1981). The real world and mathematics. Glasgow: Blackie.

Davis, R. B. (1984). Learning mathematics: The cognitive science approach to mathematics education. London: Croom Helm.

Eisenhart, M. A. (1988). The ethnographic research tradition and mathematics education research. Journal for Research in Mathematics Education, 19 (2), pp. 99-144.

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. American Psychologist, 34 (10), 906-911.

Garofalo, J., and Lester, F. (1985). Metacognition, cognitive monitoring, and mathematical performance. Journal for Research in Mathematics Education, 16 (3), pp. 163-176.

Hinsky, D., Hayes, J. and Simon, H. (1977). In Carpenter, P. and Just, M. (Eds.), Cognitive processes in comprehension, (pp. 89-106). Hillsdale, NJ: Lawrence Erlbaum.

Janvier, C. (1990). Contextualisation and mathematics for all. In Cooney, T. and Hirsch, C. (Eds.), Teaching and learning mathematics in the 1990's, (pp. 183-193) (Yearbook of the National Council of Teachers of Mathematics). Reston, Va: NCTM.

Kilpatrick, J. (1981). The reasonable ineffectiveness of research in mathematics education. For the Learning of Mathematics, 2 (2), pp. 22-28.

Lesh, R. and Akerstrom, M. (1982). Applied problem solving: Priorities for mathematics education research. In Lester, F. K. and Garofalo, J. (Eds.), Mathematical problem solving: Issues in research (pp. 117-129). Philadelphia, Pennsylvania: Franklin Institute Press.

Lesh, R., Post, T. and Behr, M. (1987). Representations and translations amoung representationsin mathematics learning and problem solving. In Janvier, C. (Ed.), Problems of representation in the teaching and learning of mathematics, (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Lester, F. (1985). Methodological considerations in research on mathematical problem-solving instruction. In Silver, E. (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives, (pp. 41-70). Hillsdale, NJ: Lawrence Erlbaum.

Lester, F., Garofalo, J. and Kroll, D. (1989). The role of metacognition in mathematical problem solving: A study of two grade seven classes. Final report to the national science foundation of NSF project MDR 85-50346.

Mason, J., Burton, L., Stacey, K. (1982). Thinking mathematically. Wokingham, England: Addison-Wesley.

McLeod, D. B. (1985). Affective issues in research on teaching mathematical problem solving. In Silver, E. (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives, (pp. 267-280). Hillsdale, NJ: Lawrence Erlbaum.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In Grouws, D. A. (Ed.), Handbook of research on mathematics teaching and learning., (pp. 575-596) New York, NY: Macmillan.

Polya, G. (1945). How to solve it. Princeton, NJ: Princeton University Press.

Polya, G. (1954). Mathematics and plausible reasoning. Princeton, NJ: Princeton University Press.

Polya, G. (1981). Mathematical Discovery (Vol 1 & 2). New York: Wiley.

Romberg, T. and Carpenter, T. (1986). Research on teaching and learning mathematics: Two disciplines of scientific enquiry. In Wittrock,

M. C. (Ed.), Handbook of research on teaching (3rd ed.). New York: Macmillan

Schoenfeld, A. (1983). Problem solving in the mathematics curriculum: A report, recommendations, and an annotated bibliography. (M.A.A. Notes#1). Washington, DC: Mathematical Association of America.

Schoenfeld, A. (1985). Mathematical problem solving. New York, NY: Academic Press.

Schoenfeld, A. (1987). Cognitive science and mathematics education : An overview. In Schoenfeld A. (Ed.), Cognitive science and mathematics education (pp. 1-31). Hillsdale, NJ: Lawrence Erlbaum.

Schoenfeld, A. (1989). Exploration of students' mathematical beliefs and behaviour. Journal for Research in Mathematics Education, 20 (4), pp. 338-355.

Schoenfeld, A. (1990). Confessions of an Accidental Theorist. Keynote address presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia. Hobart, Tasmania.

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In Grouws, D. A. (Ed.), Handbook of research on mathematics teaching and learning, (334-370). New York, NY: Macmillan.

Silver, E. A. (1979). Stuedents perceptions of relatedness among mathematical verbal problems. Journal for Research in Mathematics Education, 10 (3), pp. 195-210

Silver, E. A. (1982). Knowledge organization and mathematical problem solving. In Lester, F. K. and Garofalo, J. (Eds.), Mathematical problem solving: Issues in research (pp. 15-25). Philadelphia, Pennsylvania: Franklin Institute Press.

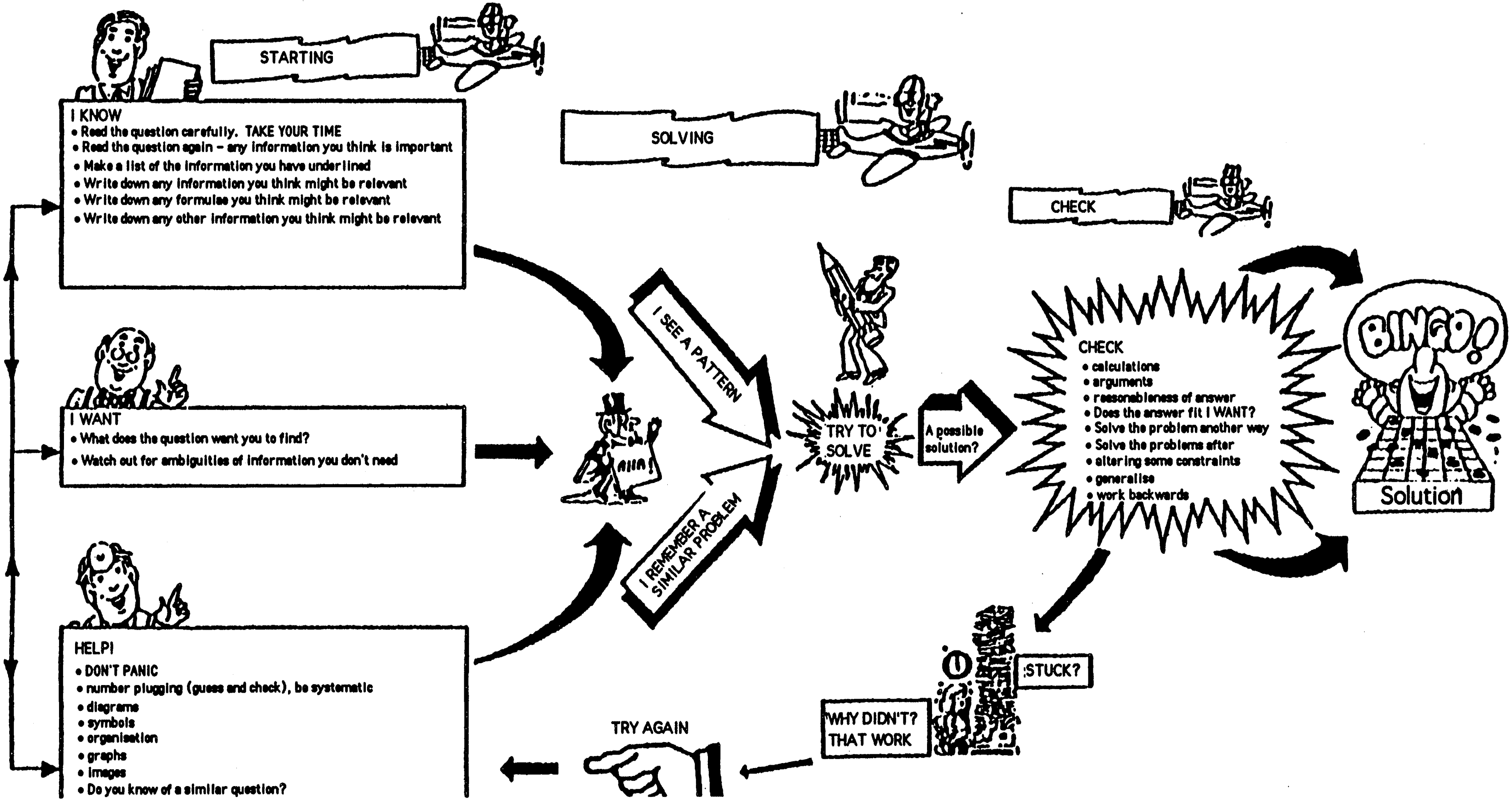
Silver, E. A. (1987). Foundations of cognitive theory and research for mathematics problem solving. In Schoenfeld A. (Ed.), Cognitive science and mathematics education, (pp. 33-60). Hillsdale, NJ: Lawrence Erlbaum.

Silver, E. and Kilpatrick, J. (1989). Testing mathematical problem solving. In Charles, R. I. and Silver, E. A. (Eds.), The teaching and assessing of mathematical problem solving (pp. 178-186). Hillsdale, NJ: Lawrence Erlbaum..

Skemp, R. R. (1971). The psychology of learning mathematics. London: Pelican.

Stenhouse, L. (1975). An Introduction to Curriculum Research and Development. London: Heinemann

Thompson, A. G. (1985). Teacher's conceptions of mathematics and the teaching of problem solving. In Silver, E. A. (Ed.), Teaching and Learning Mathematical Problem Solving: Multiple research perspectives (pp. 281-294). Hillsdale, NJ: Lawrence Erlbaum.


STARTING
I KNOW
• Read the question carefully. TAKE YOUR TIME
• Read the question again – any information you think is important
• Make a list of the information you have underlined
• Write down any information you think might be relevant
• Write down any formulae you think might be relevant
• Write down any other information you think might be relevant
I WANT
• What does the question want you to find?
• Watch out for ambiguities of information you don't need
HELP!
• DON'T PANIC
• number plugging (guess and check), be systematic
• diagrams
• symbols
• organisation
• graphs
• images
• Do you know of a similar question?
SOLVING
I SEE A PATTERN
I REMEMBER A SIMILAR PROBLEM
TRY TO SOLVE
A possible solution?
CHECK
CHECK
• calculations
• arguments
• reasonableness of answer
• Does the answer fit I WANT?
• Solve the problem another way
• Solve the problems after
• altering some constraints
• generalise
• work backwards
BINGO!
Solution
STUCK?
WHY DIDN'T THAT WORK
TRY AGAIN